SB-2734/200

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M. Sc. (Mathematics) (Sem. - II) Examination

March / April – 2011

Paper -503 : Elements of Partial Differential Equations

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. Mathematics (Sem. II)**

Name of the Subject :
☛ **503 : Elements of Partial Differential Equations**

☛ Subject Code No. : **2 7 3 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Answer all questions.
(3) Figures to the right indicate marks of the question.
(4) Follow usual notation.

1 (a) Attempt any one : **6**

(i) Obtain the direction ratio of tangent line to the curve in space.

(ii) Prove that praffian differential equation always possesses an integrating factor.

(b) Attempt any two : **8**

(i) Show that the direction cosines of the tangent at the point (x,y,z) to the conic $ax^2+by^2+cz^2=1$, $x+y+z=1$ are proportional to $(by-cz,\ cz-ax,\ ax-by)$.

(ii) Find the integral curves of the equations

$$\frac{dx}{x^2(y^3-z^3)}=\frac{dy}{y^2(z^3-x^3)}=\frac{dz}{z^2(x^3-y^3)}.$$

(iii) Find the Orthogonal trajectories on the sphere $x^2+y^2+z^2=a^2$ of its intersections with the paraboloids xy=cz, c being a parameter.

2 (a) Attempt any one : 6

(i) In usual notation discuss the method for solving the equation $P_p + Q_q = R$, where P, Q, R are functions of x, y and z.

(ii) Show that a necessary and sufficient condition there exists a relation F(u,v)=0 between two function u(x,y) and v(x,y) not involving x and y explicitly is $\frac{r(u,v)}{r(x,y)} = 0$.

(b) Attempt any two : 8

(i) Verify that the equation :

$$x(y^2 - a^2)dx + y(x^2 - z^2)dy - z(y^2 - a^2)dz = 0$$

is integrable and solve it.

(ii) Find the integral curves of the equations

$$\frac{dx}{zy - bz} = \frac{dy}{az - cx} = \frac{dz}{bx - ay}$$

and show that they are circles.

(iii) Solve the equation :

$z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ by Natani's method.

3 (a) Attempt any one : 6

(i) State and prove the necessary and sufficient condition for the pfaffian differential equation to be integrable.

(ii) Describe the method to obtain the orthogonal surfaces to the system of surface f(x,y,z) = c.

(b) Attempt any two : 8

(i) Find the general integrals of the equation $y^2 p - xyq = x(z - 2y)$.

(ii) Find the equation of the integral surface of the differential equation :

$2y(z-3)p + (2x - z)q = y(2x - 3)$ which passes through the circle z=0, $x^2 + y^2 = 2x$.

(iii) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2$, z=0.

4 (a) Attempt any one : **6**

(i) Derive the condition for the partial differential equation f(x,y,z,p,a)=0 to be compatible.

(ii) Explain Charpit's method to solve the partial differential equation f(x,y,z,p,q)=0.

(b) Attempt any two : **8**

(i) Find the solution of the equation

$$z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$$

which passes through the x-axis.

(ii) Find the complete integrals of

$2(z + xp + xz) = yp^2$ by Jacobi's method.

(iii) Find the complete integral of the equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve x=0, $z^2 = 4y$.

5 Attempt any three : **14**

(i) Find the solution of the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y.$$

(ii) Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

(iii) By separating the variables, show that the equation ∇^2, v=0 has solutions of the form $A_{\exp}(\pm nx \pm in\,y)$, where A and n are constants. Deduce that the functions of the form $v(x, y) = \sum_r A_r e^{-r\lambda x/a} \sin\frac{r\lambda y}{a}$, $x \geq 0$, $\theta \leq y \leq a$; where Ar's are constants are plane harmonic functions satisfying the conditions v(x,0)=0, v(x,a)=0, $v(x, y) \to 0$ as $x \to \infty$.

(iv) Solve the equation :

$$r - s + 2q - z = x^2 y^2.$$